Updated to 2017-18 Syllabus

CIE IGCSE
MATHS 0580

SUMMARIZED NOTES ON THE EXTENDED SYLLABUS

TABLE OF CONTENTS

1. NUMBER

- **Natural numbers:**
  - used for counting purposes
  - made up off all possible rational & irrational numbers
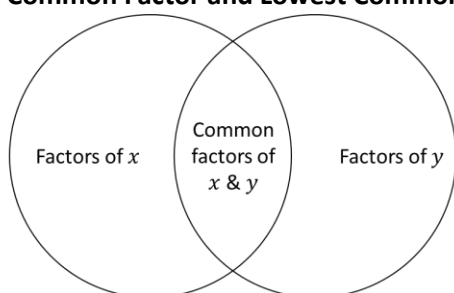- **Integer: a** whole number
- **Prime numbers:**
  - divisible only by itself and one
  - 1 is not a prime number
- **Rational numbers:** can be written as a fraction
- **Irrational numbers:** cannot be written as a fraction e.g. π

1.1 HCF and LCM

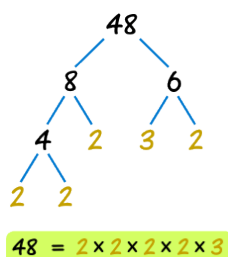- **Highest Common Factor and Lowest Common Multiple:**

  - HCF = product of common factors of x and y
  - LCM = product of all items in Venn diagram
- **Prime Factorization:** finding which prime numbers
  - multiply together to make the original number

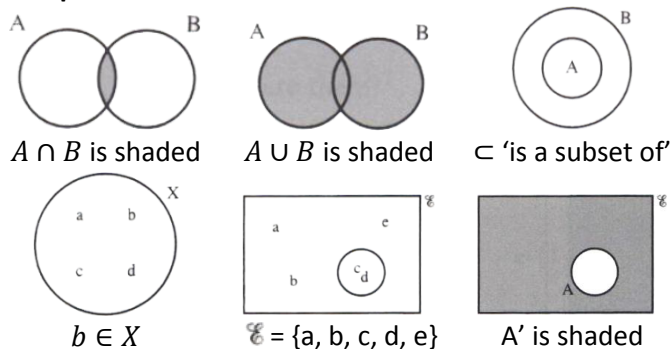$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

1.2 Sets

- Definition of sets e.g.
  - $A = \{x : x \text{ is a natural number}\}$
  - $B = \{(x, y) : y = mx + c\}$
  - $C = \{x : a \leq x \leq b\}$
  - $D = \{a, b, c, \dots\}$

Notation:

- $n(A)$ = no. of elements in A
- $\in$ = …is an element of…
- $\notin$ = …is not an element of…
- A' = compliment of set A
- $\emptyset$ or { } = empty set
- $\mathscr{E}$ = Universal set
- $A \cup B$ = union of A and B
- $A \cap B$ = intersection of A and B
- $A \subseteq B$ = A is a subset of B
- $A \subset B$ = A is a proper subset of B
- $A \nsubseteq B$ = A is not a subset of B
- $A \not\subset B$ = A is not a proper subset of B

Set representations:

$A \cap B$ is shaded $A \cup B$ is shaded $\subset$ 'is a subset of'

$b \in X$ $\mathscr{E}$ = {a, b, c, d, e}   A' is shaded

1.3 Indices

- $n(A)$ = no. of elements in A
- $\in$ = …is an element of…
- $\notin$ = …is not an element of…
- A' = compliment of set A
- $\emptyset$ or { } = empty set
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- $A \cup B$ = union of A and B
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- $A \nsubseteq B$ = A is not a subset of B
- $A \not\subset B$ = A is not a proper subset of B

Standard form:

- $10^4 = 10000$
- $10^3 = 1000$
- $10^2 = 100$
- $10^1 = 10$
- $10^0 = 1$

$10^{-1} = 0.1$
$10^{-2} = 0.01$
$10^{-3} = 0.001$
$10^{-4} = 0.0001$
$10^{-5} = 0.00001$

Limits of accuracy:

- The degree of rounding of a number
  - E.g. 2.1 to 1 d.p. $2.05 \leq x < 2.15$

1.4 Ratio & Proportion

- **Ratio:** used to describe a fraction
  - e.g. 3 : 1
- **Foreign exchange:** money changed from one currency to another using proportion
  - E.g. Convert $22.50 to Dinars
    $1 : 0.30KD
    $22.50 : 6.75KD
- **Map scales:** using proportion to work out map scales
  - 1km = 1000m
  - 1m = 100cm
  - 1cm = 10mm

- **Direct variation:** y is proportional to x
$$y \propto x \qquad y = kx$$
- **Inverse variation:** y is inversely proportional to x
$$y \propto \frac{1}{x} \qquad y = \frac{k}{x}$$

1.5 Percentages
- **Percentage:**
  - Convenient way of expressing fractions
  - Percent means per 100
- **Percentage increase or decrease:**
$$Percentage\ increase = \frac{Actual\ Increase}{Original\ Amount}$$
- **Simple interest:**
$$I = \frac{PRT}{100}$$
$P = Principal \qquad R = Rate\ of\ Interest \qquad T = Period\ of\ Time$
- **Compound interest:**
$$A = P\left(1 + \frac{R}{100}\right)^n$$
$P = Principal \qquad R = Rate\ of\ Interest \qquad n = Period\ of\ Time$

1.6 Speed, Distance & Time
$$Speed = \frac{Distance}{Time}$$
$$Average\ Speed = \frac{Total\ Distance}{Total\ Time}$$
- **Units of speed:** km/hr m/s
- **Units of distance:** km m
- **Units of time:** hr sec
$$km/hr \times \frac{5}{18} = m/sec$$
$$m/sec \times \frac{18}{5} = km/hr$$

2. ALGEBRA & GRAPHS

2.1 Factorisation
- **Common factors:**
$$3x^2 + 6x$$
$$3x(x + 2)$$
- **Difference of two squares:**
$$25 - x^2$$
$$(5 + x)(5 - x)$$
- **Group factorization:**
$$4d + ac + ad + 4c$$
$$4(d + c) + a(c + d)$$
$$(4 + a)(c + d)$$

- **Trinomial:**
$$x^2 + 14x + 24$$
$$x^2 + 12x + 2x + 24$$
$$x(x + 12) + 2(x + 12)$$
$$(x + 2)(x + 12)$$

2.2 Quadratic Factorization
- **General equation:**
$$ax^2 + bx + c = 0$$
- **Solve quadratics by:**
  - Trinomial factorization
  - Quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- When question says "give your answer to two decimal places", **use formula!**

2.3 Simultaneous Equations
- Simultaneous linear equations can be solved either by substitution or elimination
- Simultaneous linear and non-linear equations are generally solved by substitution as follows:
  - Step 1: obtain an equation in one unknown and solve this equation
  - Step 2: substitute the results from step 1 into the linear equation to find the other unknown
- The points of intersection of two graphs are given by the solution of their simultaneous equations

2.4 Inequalities
- Solve like equations
- Multiplying or dividing by negative ⇒ switch sign
$$\frac{y}{-3} \geq -7$$
$$y \leq -7 \times -3$$
$$y \leq 21$$
- When two inequalities present, split into two
$$x < 3x - 1 < 2x + 7$$
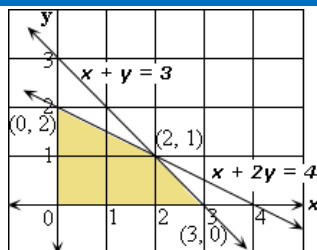$$x < 3x - 1 \qquad\qquad 3x - 1 < 2x + 7$$
$$x > -\frac{1}{2} \qquad\qquad\qquad x < 8$$

2.4 Linear Programming
- For strict inequalities ($<, >$) use broken line
- For non-strict inequalities ($\leq, \geq$) use solid line

- Steps to solve:
  - Interpret $y = mx + c$
  - Draw straight line graphs
 - Shade
  - Solve

2.5 Sequences

- **Linear sequences:** Find common difference e.g. 3 then multiply by n and work out what needs to be added
- **Quadratic sequences:**
  - Format: $an^2 + bn + c$

$$a + b + c =$$
$$3a + b =$$
$$2a =$$

  - Work out the values and then place into formula to work out nth term formula
- **Geometric progression:** sequence where term has been multiplied by a constant to form next term

$$nth\ term\ of\ G.P. = ar^{(n-1)}$$

  - a = 1st term r = common difference

2.6 Distance-Time Graphs
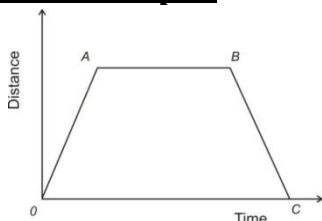
- From O to A : Uniform speed
- From B to C : Uniform speed (return journey)
- From A to B : Stationery (speed = 0)
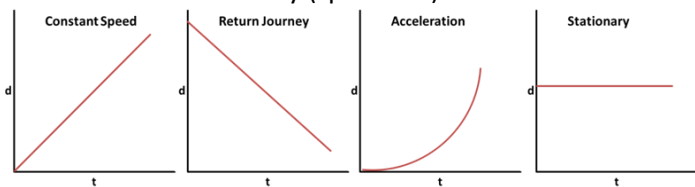
- Gradient = speed

2.7 Speed-Time Graphs
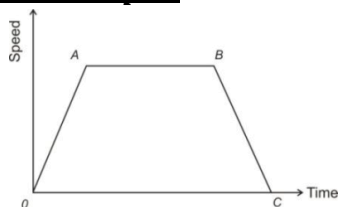
- From O to A : Uniform speed
- From A to B : Constant speed (acceleration = 0)
- From B to C : Uniform deceleration / retardation
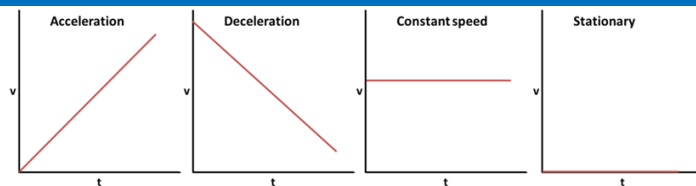
- Area under a graph = distance travelled.
- Gradient = acceleration.
- If the acceleration is negative, it is called deceleration or retardation. (moving body is slowing down.)

2.8 Functions

- **Function notation:**
  - $f: x \rightarrow 2x - 1$
  - Function f such that x maps onto $2x - 1$
- **Composite function:** Given two functions $f(x)$ and $g(x)$, the composite function of f and g is the function which maps x onto $f(g(x))$
- $f(2)$
  - Substitute $x = 2$ and solve for $f(x)$
- $fg(x)$
  - Substitute $x = g(x)$
- $f^{-1}(x)$
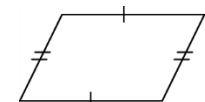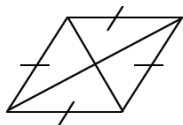 - Let $y = f(x)$ and make x the subject

3. GEOMETRY

3.1 Triangles
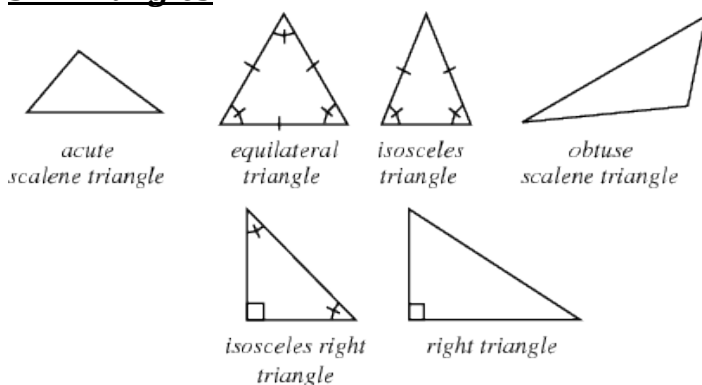
3.2 Quadrilaterals

- **Rectangle:** Opposite sides parallel and equal, all angles 90°, diagonals bisect each other.

- **Parallelogram :** Opposite sides parallel and equal, opposite angles equal, diagonals bisect each other
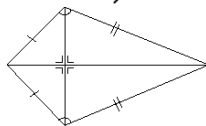
- **Rhombus:** A parallelogram with all sides equal, opposite angles equal, diagonals bisect each other

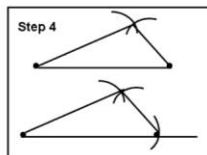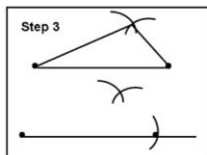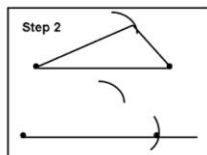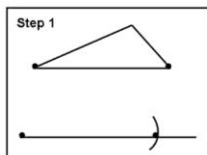- **Trapezium:** One pair of sides parallel
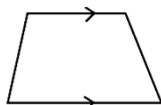
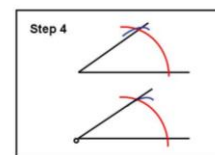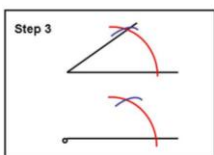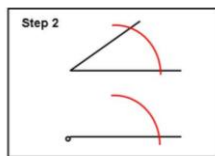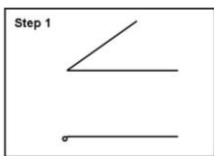- **Kite:** Two pairs of adjacent sides equal, diagonals meet at right angles bisecting one of them
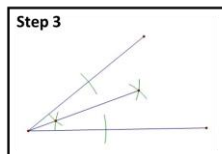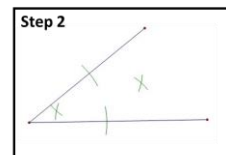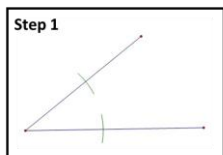
3.3 Construction

- **Constructing triangles:**

- **Perpendicular bisector:**

- **Angle bisector:**

3.4 Symmetry

- A **line of symmetry** divides a two-dimensional shape into two congruent (identical) shapes.
- A **plane of symmetry** divides a three-dimensional shape into two congruent solid shapes.

- The number of times shape fits its outline during a complete revolution is called the order of **rotational symmetry**.
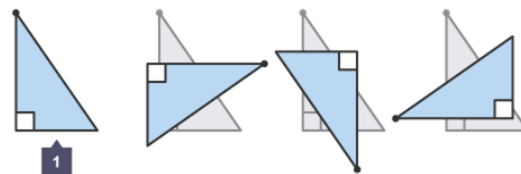
| Shape	Number of Lines of Symmetry	Rotational Symmetry Order
Square	4	4
Rectangle	2	2
Parallelogram	0	2
Rhombus	2	2
Trapezium	0	1
Kite	1	1
Equilateral triangle	3	3
Regular hexagon	6	6

- **Properties of circles:**
  - Equal chords are equidistant from the centre
  - The perpendicular bisector of a chord passes through the centre
  - Tangents from an external point are equal in length

3.5 Polygons

- Sum of angles at a point = 360
- Angles on a straight line = 180
- Sum of angles in a triangle =180
- For regular polygon
  - External angles $= \frac{360}{n}$
  - Internal angles $= 180 - \frac{360}{n}$
- For irregular polygon:
  - Sum of exterior angles =360
  - Sum of interior angles =180(n-2)
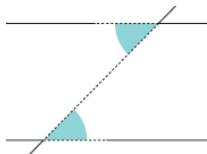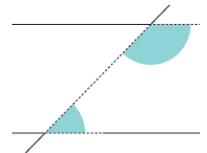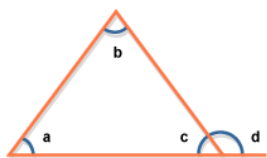- Vertically opposite angles
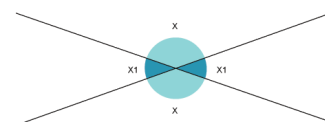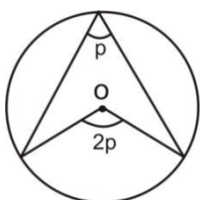
- Corresponding angles
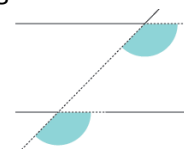
- Alternate angles

- Co-interior angles
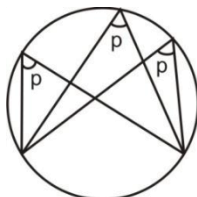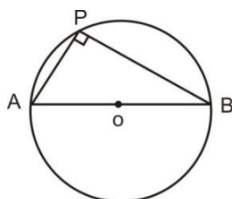

- Exterior angle=sum of interior opposite ∠

3.6 Circle Theorem
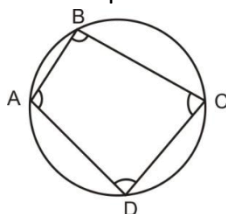

Angle at centre = twice angle on circumference
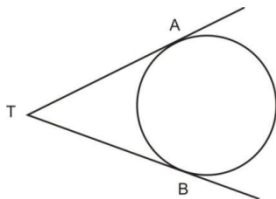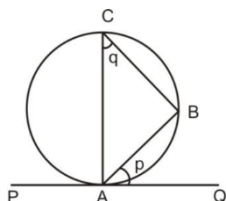

Angle subtended by same arc at circumference are equal

Angles in semicircle are 90°

Opposite angles in a cyclic quadrilateral = 180°

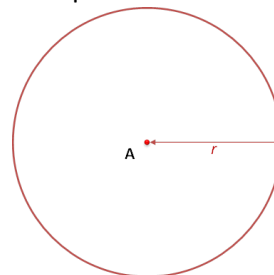Tangents from one point are equal ∠ between tangent and radius is 90°

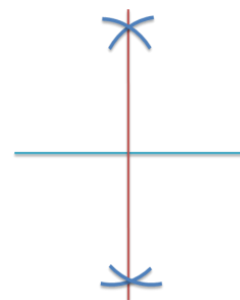

Alternate segment theorem

3.7 Loci

- The locus of points equidistant from a point is a circle

- The locus of points equidistant between two point is a perpendicular bisector

- The locus of points equidistant between two lines is an angle bisector
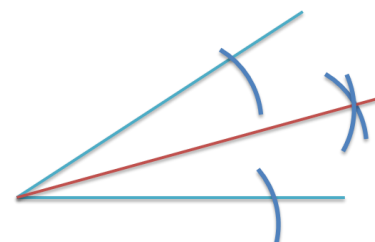
- The locus of points equidistant (along) from a line is a parallel line
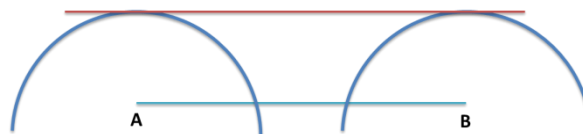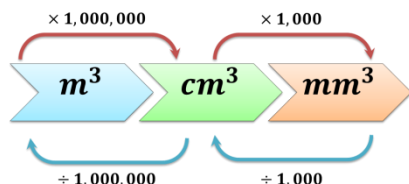

4. MENSURATION

4.1 Area

- Parallelogram $= b \times h$ OR $ab \sin\theta$
- Triangle$= \frac{1}{2}b \times h$
- Trapezium$= \frac{1}{2}(a + b)h$
- Circle$= \pi r^2$
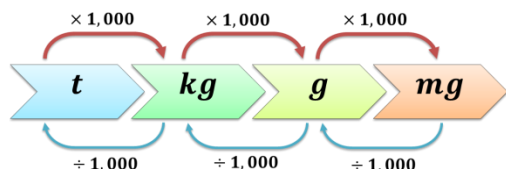- Sector$= \pi r^2 \times \frac{\theta}{360}$

4.2 Volume and Surface Area

- Cylinder
  - $Curved\ surface\ area = 2\pi rh$
  - $Volume = \pi r^2 h$
- Cone
  - $Curved\ surface\ area = \pi rl$
  - $Volume = \frac{1}{3}(\pi r^2 h)$
- Sphere
  - $Surface\ area = 4\pi r^2$
  - $Volume = \frac{4}{3}\pi r^3$
- Hemisphere
  - $Surface\ area = 2\pi r^2$
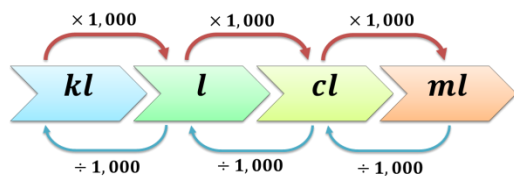 - $Volume = \frac{2}{3}\pi r^3$

4.3 Units

- **Volume:**

- **Mass:**

- **Capacity:**

- **Connecting volume and capacity:**
  - $1ml = 1cm^3$
  - $1kl = 1m^3$
- **Density** $= \frac{Mass}{Volume}$
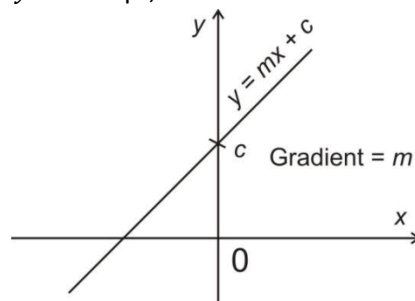
5. COORDINATE GEOMETRY

5.1 Graphs

- **Gradient of a Straight Line:**
$$Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

- **Equation of Line:**
$$y = mx + c$$
  - Find the gradient, m
  - Find the y-intercept, c

Gradient = m

- **Midpoint of Graph:**
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- **Length between two points:**
$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

5.2 Sketching Graphs
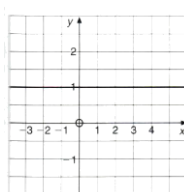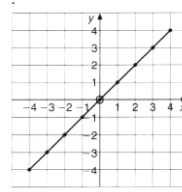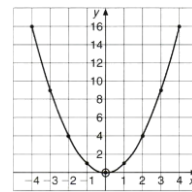
$f(x) = 1$ $f(x) = x$ $f(x) = x^2$

$f(x) = x^3$ $f(x) = \frac{1}{x}$ $f(x) = \frac{1}{x^2}$

6. TRIGONOMETRY

6.1 Bearings

- **The bearing of a point B from another point A is:**
  - An angle measured from the north at A.
  - In a clockwise direction.
  - Written as three-figure number (i.e. from 000° to 360°)
- e.g. The bearing of B from A is 050°

6.2 Pythagoras Theorem

- To find hypotenuse
  - $a^2 + b^2 = c^2$
- To find one of the shorter sides
  - $a^2 = c^2 - b^2$
  - $b^2 = c^2 - a^2$

- **Angle of elevation:**
  - Angle above the horizontal line.
- **Angle of depression:**
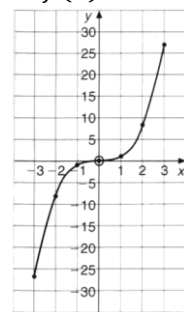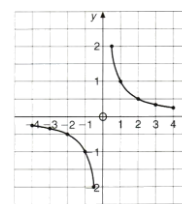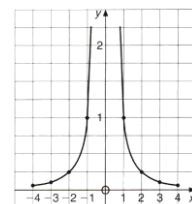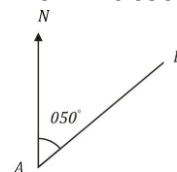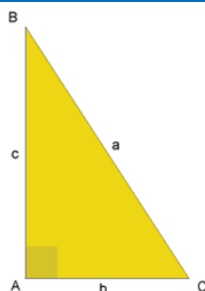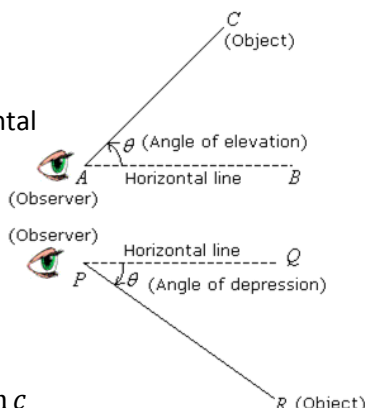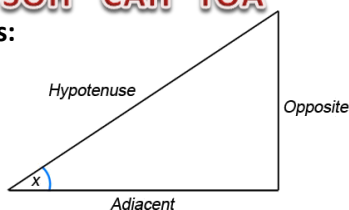 - Angle below the horizontal line.

- **Area of a triangle:** $\frac{1}{2} ab \sin c$

6.3 Ratios

SOH   CAH   TOA

- **Right angled triangles:**
  - $\sin x = \frac{opposite}{hypotenuse}$
  - $\cos x = \frac{adjacent}{hypotenuse}$
  - $\tan x = \frac{opposite}{adjacent}$

6.4 Sine & Cosine Rules

- **Sine rule:**

$$\frac{a}{\sin a} = \frac{b}{\sin b} = \frac{c}{\sin c}$$

  - One pair of information needed
- **Cosine rule**
  - To find the angle given 3 sides

$$\cos a = \frac{b^2 + c^2 - a^2}{2bc}$$

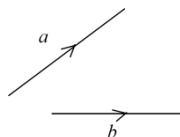 - To find side given angle and two sides

$$a^2 = b^2 + c^2 - 2bc \cos a$$

7. MATRICES & TRANSFORMATION

7.1 Vector

- **A vector quantity** has both magnitude and direction.
  - E.g. Vectors a and b represented by the line segments can be added using the parallelogram rule or the nose-to-tail method.

- **Multiplication by a scalar:**
  - A scalar quantity has a magnitude but no direction
  - The negative sign reverses the direction of the vector
- **Column vector:**
  - Top number is the horizontal component and bottom number is the vertical component $\begin{pmatrix} x \\ y \end{pmatrix}$
- **Parallel vectors:**
  - Vectors are parallel if they have the same direction
  - In general the vector $k\begin{pmatrix} a \\ b \end{pmatrix}$ is parallel to $\begin{pmatrix} a \\ b \end{pmatrix}$
- **Modulus of a vector:**
  - In general, if $x = \begin{pmatrix} m \\ n \end{pmatrix}$, $|x| = \sqrt{(m^2 + n^2)}$

7.2 Matrices

- **Addition:**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix}$$

- **Multiplication by scalar**

$$k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

- **Multiplication by vector:**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{pmatrix}$$

  - You can only multiply if no. of columns in left equals to no. of rows in right
- **Determinant:**
  - Determinant = leading diagonal – secondary diagonal

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad |A| = (ad) - (bc)$$

- **Inverse:**
  - To work out inverse, switch leading diagonal, negate secondary diagonal, multiply by $\frac{1}{|a|}$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad A^{-1} = \frac{1}{(ad-bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

7.3 Transformation

- **Reflection (M):**
  - When describing a reflection, the position of the mirror line is essential.
- **Rotation (R):**
  - To describe a rotation, the centre of rotation, the angle of rotation and direction of rotation are required.
  - A clockwise rotation is negative and an anticlockwise rotation is positive.
- **Translation (T):**
  - When describing a translation it is necessary to give the translation vector $\begin{pmatrix} x \\ y \end{pmatrix}$

- **Enlargement (E):**
  - To describe an enlargement, state the scale factor, K and the centre of enlargement

$$Scale\ factor = \frac{length\ of\ image}{length\ of\ object}$$

$$Area\ of\ image = K^2 area\ of\ object$$

  - If K > 0, both object and image lie on same side of the centre of enlargement.
  - If K < 0, object and image lie on opposite side of the centre of enlargement.

7.4 Transformation by Matrices

- **Reflection:**
  - $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Reflection in the $x - axis$
  - $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ Reflection in the $y - axis$
  - $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reflection in the line $y = x$
  - $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ Reflection in the line $y = -x$
- **Enlargement:**
  - $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ where k=scale factor and centre of enlargement = (0,0)
- **Rotation:**
  - $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Rotation 90° anticlockwise, centre (0,0)
  - $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ Rotation 90° clockwise, centre (0,0)
  - $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ Rotation 180° clockwise/ anticlockwise, centre (0,0)

8. PROBABILITY

- Probability is the study of chance, or the likelihood of an event happening.

$$Probability\ of\ an\ event = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

- If probability = 0, the event is impossible and if probability =1, the event is certain to happen
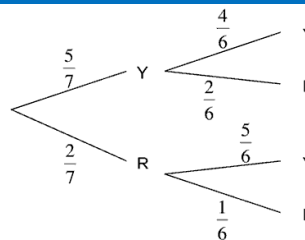- All probabilities lie between 0 and 1.

8.1 Events

Exclusive events:
- Two events are exclusive if they cannot occur at the same time.
- **The OR Rule:**
  - For exclusive events A and B
  - p(A or B) = p(A) + p(B)

Independent events:
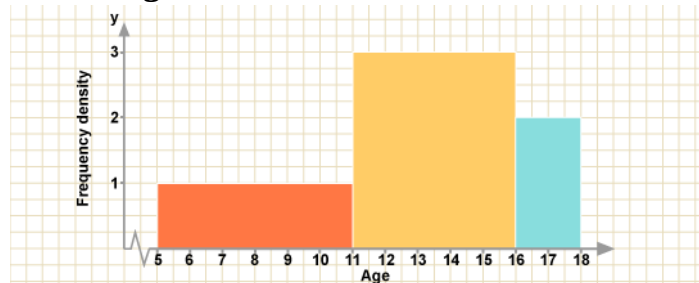- Two events are independent if occurrence of one is unaffected by occurrence of other.
- **The AND Rule:**
  - p(A and B) = p(A) × p(B)

9. STATISTICS

9.1 Histograms

- A histogram displays the frequency of either continuous or grouped discrete data in the form of bars.
- The bars are joined together.
- The bars can be of varying width.
- The frequency of the data is represented by the area of the bar and not the height.
- When class intervals are different it is the area of the bar which represents the frequency not the
- height
- Instead of frequency being plotted on the vertical axis, frequency density is plotted**.**
- **Class width** = Interval
- **Frequency density** = Height

$$Frequency = Class\ width \times Frequency\ density$$

9.2 Averages

- **Mean:**

$$\frac{Sum\ of\ values}{number\ of\ values}$$

- **Median:**
  - The middle value when the data has been written in ascending or descending order

o Odd no. of values $\frac{5+1}{2} = 3rd\ value$

o Even no. of values $\frac{6+1}{2} = 3.5th\ value$

(add two values divide by 2)

- **Mode:**
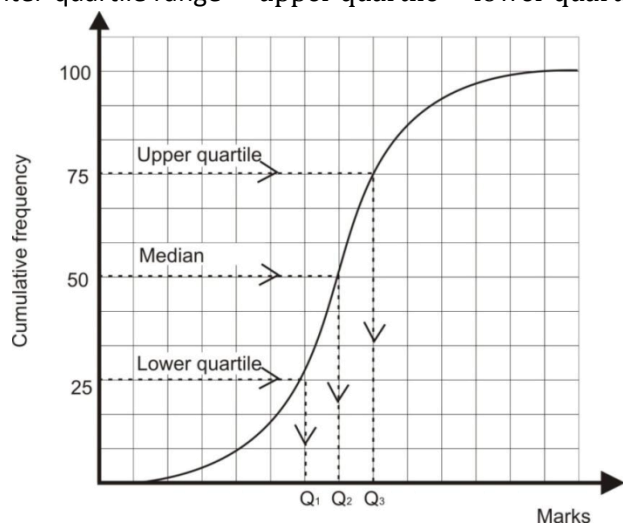 o Most frequently occurring value
- **Range:**
  o Difference between highest and lowest values
- **Estimated mean of grouped data:**
  o Work out midpoints of each group and multiply by frequency
  o Divide by number of values

9.3 Cumulative Frequency

- Cumulative frequency is the total frequency up to a given point.
- Inter-quartile range $=$ upper quartile $-$ lower quartile

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