MATH REVISION BOOKLET

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Tuesday, March 6, 2018

Note: Although the chapters in this booklet align with those from the textbook, the sub-topics in this booklet corresponds to one or more sub-topics in the textbook.

CONTENTS

1 LOGARITHMS

SIMPLIFYING EXPONENTS AND SURDS 1A

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$

$a^{\frac{n}{m}} = \sqrt[m]{a^n}$

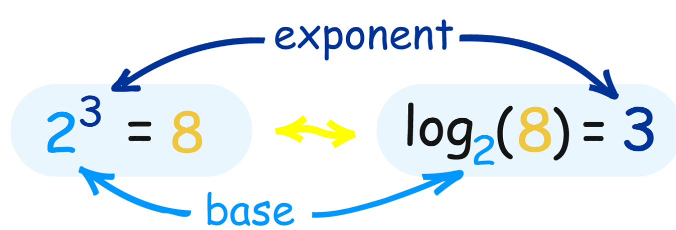
$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$\frac{1}{\sqrt{a}} = \frac{1 \times \sqrt{a}}{\sqrt{a} \times \sqrt{a}} = \frac{\sqrt{a}}{a}$

LOGARITHMS 1B

$log_a n = x$ means that $a^x = n$



$log_a 1 = 0$ since $a^0 = 1$

$log_a a = 1$ since $a^1 = a$

$log_a xy = log_a x + log_a y$

$log_a \frac{x}{y} = log_a x - log_a y$

let $log_a x = b$, $log_a y = c$,
so that $a^b = x$ and $a^c = y$
$a^b \times a^c = \underline{a^{b+c} = xy}$
$log_a xy = b + c = log_a x + log_a y$

$log_a(x^k) = k\ log_a x$

$log_a x = \frac{log_b x}{log_b a}$

let $log_a x = m$, so that $a^m = x$
$log_b(a^m) = log_b(x)$
$m\ log_b a = log_b x$
$log_a x \times log_b a = log_b x$

2 QUADRATIC FUNCTION

QUADRATIC EQUATION 2A

A quadratic: $ax^2 + bx + c$

Factorize:

$(\alpha x + a)(\beta x + b)$ in which $\alpha\beta = a$; $a\beta + \alpha b = b$; and $a + b = c$

$$x^2 - y^2 = (x + y)(x - y)$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$(x - y)^2 = x^2 + y^2 - 2xy$$

Complete the Square: add a number to obtain a square, and deduct later

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ — discriminant

- > 0 …… two roots
- $= 0$ …… 1 root
- < 0 …… no solution

$$ax^2 + bx + c = 0$$
$$x^2 + bx/a = -c/a$$
$$x^2 + bx/a + (b/2a)^2 = (b/2a)^2 - c/a$$
$$(x + b/2a)^2 = (b^2 - 4ac)/4a^2$$
$$x + b/2a = \pm\sqrt{(b^2 - 4ac)}/2a$$
$$x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$$

sum of roots = $\alpha + \beta = -b / a$

product of roots = $\alpha\beta = c / a$

$$(x-\alpha)(x-\beta) = x^2 + bx + c$$
$$x - (\alpha+\beta)x + \alpha\beta = x^2 + bx + c$$
$$b = -(\alpha+\beta)$$
$$c = \alpha\beta$$

3 IDENTITIES AND INEQUALITIES

DIVIDING POLYNOMIAL 3A

Polynomials can be divided by (ax ± b), much like a regular division

Factor Theorem: if f(p) = 0 for f(x), then

(x-p) is a factor of f(x)

e.g. (x-1) is factor of $f(x) = 4x^3 - 3x^2 - 1$

because $f(1) = 4(1^3) - 3(1^2) - 1 = 0$

Remainder Theorem: f(x) divided by (ax-b)

has a remainder of f(b/a)

$$5x^2$$

$$x-4 \overline{)5x^3 - x^2 + 0x + 6}$$

$$-\left(5x^3 - 20x^2\right)$$

$$19x^2 + 0x + 6$$

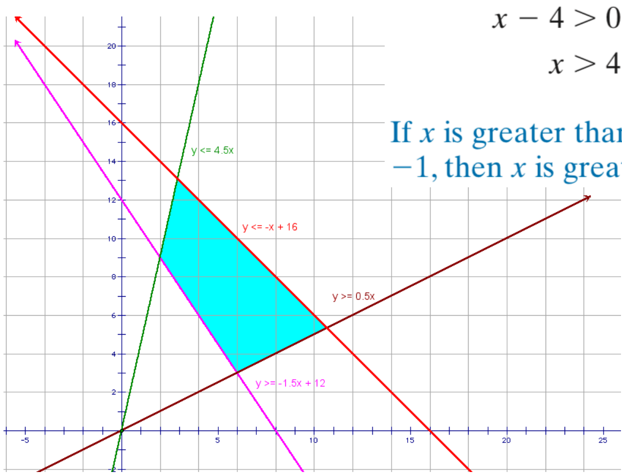
SOLVING QUADRATIC INEQUALITIES 3B

1. Solve the Quadratic
2. Sketch the Graph or Otherwise
3. Find Appropriate Range

$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x - 4 = 0$$
$$x + 1 = 0$$

$$x^2 - 3x - 4 > 0$$
$$(x-4)(x+1) > 0$$
$$x - 4 \overset{?}{>} 0$$
$$x + 1 \overset{?}{>} 0$$

Or: linear programming



$$x^2 - 3x - 4 > 0$$
$$(x-4)(x+1) > 0$$

$x - 4 > 0$ and $x + 1 > 0$	$x - 4 < 0$ and $x + 1 < 0$
$x > 4$ $x > -1$	$x < 4$ $x < -1$
If x is greater than 4 and greater than -1, then x is greater than 4.	If x is less than 4 and less than -1, then x is less than -1.

The solution set is $\{x : x > 4 \text{ or } x < -1\}$.

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"The roots of education are bitter, but the fruit is sweet."

—Aristotle